On Some Associated Polynomials Defined Through Generating Functions

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DOI:10.37648/ijrst.v15i04.003

[1] *Received: 10/09/2025; Accepted: 09/10/2025; Published: 22/10/2025*

Abstract

Gould [14] considered a class of genelized humbert polynomials and srivastava [23] introduced a class of generalized Hermite Polynomials, then Chandel [6],[7] considered a generating function. In the present paper we introduce associated polynomials for the polynomials difined by [8],[9],[10] and [11], we also introduce polynomials related to the polynomials defined by [1],[2],[12] and [22].

2020 Mathematics Subject Classification : 33C50

Keywords : *Humbert Polynomials; Hermite polynomials; multivariable generalized polynomials and Laguerre polynomials.*

1. Introduction

Gould [14] considered a class of generalized Humbert polynomials defined by

(1.1) $$(C - mxt + yt^m)^p = \sum_{n=0}^{\infty} P_n(m, x, y, p, C)t^n.$$

where *m* is positive integer and other parameters are unrestricted in general.

Panda [19] introduced polynomials $\{\frac{g_n^c(x,r,s)}{n} = 0,1, \ldots\}$ generated by

(1.2) $$(1-t)^{-C} G\left(\frac{xt^s}{(1-t)^r}\right) = \sum_{n=0}^{\infty} g_n^c(x, r, s)t^n ,$$

where *c* is arbitrary, *r* is any integer positive or negative and *s*=1,2,3,… .

On the other hand Srivastava [23] considered a class of generalized Hermite polynomials defined by the generating function

(1.3) $$\sum_{n=0}^{\infty} \gamma_m^{(x)} \frac{t^n}{n!} = G(mxt - t^m),$$

and in (1.2)

where

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[1] How to cite the article: Chauhan S.S., Srivastava H., Chauhan S.S. (October, 2025); On Some Associated Polynomials Defined Through Generating Functions; *International Journal of Research in Science and Technology*; Vol 15, Issue 4; 17-30, DOI: http://doi.org/10.37648/ijrst.v15i04.003

(1.4) $G(z) = \sum_{n=0}^{\infty} \gamma_n\, z^n,$

To unify the study of the three general classes of polynomials defined through (1.1), (1.2) and (1.3), Chandel [6] considered a generating function

(1.5) $(C - mxt + yt^m)^p G\left[\frac{r^r x t^s}{(C-mxt+yt^m)^r}\right] = \sum_{n=0}^{\infty} R_n^p(m,x,y,r,s,C)t^n,$

where *m,s* are positive integers, other parameters are unrestricted in general and $G(z)$ is given by (1.4). He also discussed the interesting special case where $\gamma_n = \frac{(-1)^n}{n!}$.

(1.6) $(C - mxt + yt^m)^p exp\left[\frac{x r^r t^s}{(C-mxt+yt^m)^r}\right] = \sum_{n=0}^{\infty} C_n^p(m,x,y,r,s,C)t^n.$

Further to study associated polynomials, Chandel and Bhargava [4] considered the special case of (1.5) for $\gamma_n = \frac{(q)_n}{n!}$, $s = r$, $m=2$.

(1.7) $(C - 2xt + yt^2)^p exp\left[1 - \frac{x r^r t^s}{(C-2xt+yt^2)^r}\right]^{-q}$

$$= \sum_{n=0}^{\infty} B_n^{(p,q)}(2,x,y,r,s,C)t^n.$$

Prompted by (1.5), Chandel and Dwivedi [5] considered another generating function :

(1.8) $(C - mxt + yt^m)^p G\left[\frac{y r^r t^s}{(C-mxt+yt^m)^r}\right] = \sum_{n=0}^{\infty} R_n^p(m,x,y,r,s,C)t^n,$

different from (1.5), He also considered their special cases when $\gamma_n = \frac{(-1)^n}{n!}$.

(1.9) $(C - mxt + yt^m)^p exp\left[-\frac{y r^r t^s}{(C-mxt+yt^m)^r}\right] = \sum_{n=0}^{\infty} E_n^p(m,x,y,r,s,C)t^n,$

and when $\gamma_n = \frac{(q)_n}{n!}$.

(1.10) $(C - mxt + yt^m)^p \left[1 - \frac{y r^r t^s}{(C-mxt+yt^m)^r}\right]^{-q}$

$$= \sum_{n=0}^{\infty} E_n^{(p,q)}(m,x,y,r,s,C)t^n,$$

but in no case he tried to introduce associated polynomials.

In this paper, we shall introduce associated polynomials for the polynomicals defined by (1.10) and [8],[9],[10].

Further Dwivedi [11] considered another generating function

(1.11) $(C - mxt + yt^m)^p G\left[\frac{r^r z t^s}{(C-mxt+yt^m)^r}\right] = \sum_{n=0}^{\infty} C_n^p(m,x,y,r,s,C)t^n,$

with z independent of x and y. He also considered its special cases for $\gamma_n = \frac{(-1)^n}{n!}$ and $\gamma_n = \frac{(q)_n}{n!}$.

To study associated polynomials of class (1.11) Chandel and Dwivedi [3] considered by the case of $\gamma_n = \frac{(-1)^n}{n!}$, s=r and m =2.

(1.12) $(C-2xt+yt^2)^p exp\left[1-\frac{r^r zt^s}{(C-2xt+yt^2)^r}\right] = \sum_{n=0}^{\infty} E_n^p(2,x,y,r,s,C)t^n.$

Now in the present paper we shall also introduce polynomials related of the polynomials [1],[2],[12],[20] and [22] for which $\gamma_n = \frac{(q)_n}{n!}$, s=r and m =2.

(1.13) $(C-2xt+yt^2)^p\left[1-\frac{r^r zt^s}{(C-2xt+yt^2)^r}\right]^{-q} = \sum_{n=0}^{\infty} M_n^{(p,q)}(2,x,y,r,s,C)t^n.$

2. **The Polynomials** $\left\{\frac{A_n^{p,q}(x,y,r,C)}{n} = \mathbf{0},\mathbf{1},\mathbf{2},\dots\right\}$, For s=r, m=2. (1.10) defines

(2.1) $(C-2xt+yt^2)^p\left[1-\frac{r^r yt^s}{(C-2xt+yt^2)^r}\right]^{-q} = \sum_{n=0}^{\infty} F_n^{(p,q)}(2,x,y,r,s,C)t^n,$

Consider

(2.2) $\sum_{k=0}^{\infty} A_k^{(p,q)}(x,y,r,C)\,F_{n-k}^{(p-k,q)}(2,x,y,r,r,C) = 0$

and

(2.3) $A_0^{(p,q)}(x,y,r,C) = 1.$

Therefore

1=$\sum_{n=0}^{\infty} t^n \sum_{k=0}^{n} A_k^{(p,q)}\,(x,y,r,C)\,F_{n-k}^{(p-k,q)}(2,x,y,r,r,C)$

=$\sum_{n=0}^{\infty}\sum_{k=0}^{n} A_k^{(p,q)}\,(x,y,r,C)\,F_{n-k}^{(p-k,q)}(2,x,y,r,r,C)t^{n+k}$

=$\sum_{k=0}^{n} A_k^{(p,q)}\,(x,y,r,C)t^k\ \sum_{n=0}^{\infty} F_{n-k}^{(p-k,q)}\,(2,x,y,r,r,C)t^n$

=$\sum_{k=0}^{n} A_k^{(p,q)}\,(x^j,y,r,C)t^{k_j}\,(C-2xt+yt^2)^{p-q}\left[1-\frac{r^r yt^s}{(C-2xt+yt^2)^r}\right]^{-q}$

=$(C-2xt+yt^2)^p\left[1-\frac{r^r yt^s}{(C-2xt+yt^2)^r}\right]^{-q}\sum_{k=0}^{n} A_k^{(p,q)}\,(x,y,r,C)\frac{t^k}{(C-2xt+yt^2)^k}$

Thus

(2.4) $(C-2xt+yt^2)^{-p}\left[1-\frac{r^r yt^s}{(C-2xt+yt^2)^r}\right]^{q}$

$$= \sum_{k=0}^{\infty} A_k^{(p,q)}\,(x,y,r,C)\left[\frac{t}{(C-2xt+yt^2)^k}\right]^k$$

Take $t/(C-2xt+yt^2) = w$

Therefore

(2.5) $\sum_{n=0}^{\infty} A_n^{(p,q)}\,(x,y,r,C)w^n = \left(\frac{w}{t}\right)^p[1-r^r yw^r]^q$

where

(2.6) $t = \frac{2xw+1+\sqrt{(2xw+1)^2-4ycw^2}}{2xy}$

Thus

(2.7) $\sum_{n=0}^{\infty} A_n^{(p,q)}(x,y,r,C)w^n = \left[\frac{2xw+1+\sqrt{(2xw+1)^2-4ycw^2}}{2yw^2}\right]^{-p}[1-r^r y w^r]^q$

3. Applications of Generating Relation. An appeal to generating relation (2.7) gives

(3.1) $A_n^{(p+p',q+q')}(x,y,r,C) = \sum_{k=0}^{\infty} A_k^{(p,q)}(x,y,r,C)A_{n-k}^{(p',q')}(x,y,r,C),$

which can be generalized in the following form:

(3.2) $A_n^{(p_1+\cdots+p_m,q_1+\cdots+q_m)}(x,y,r,C)$

$$= \sum_{i_1+\cdots+i_m=n}^{\infty} A_{i_1}^{(p,q)}(x,y,r,C)A_{i_m}^{(p_m,q_m)}(x,y,r,C)$$

An appeal to (2.7), further shows that

(3.3) $A_n^{(p,q+1)}(x,y,r,C) = A_n^{(p,q)}(x,y,r,C) - r^r y A_{n-r}^{(p,q)}(x,y,r,C)$

(3.4) $A_n^{(p,q)}(x,y,r,C) = \sum_{k=0}^{[n/r]} A_{n-rk}^{(p,q)}(x,y,r,C)(r^r y)^k,$

(3.5) $A_n^{(p,q-q')}(x,y,r,C) = \sum_{k=0}^{n} A_{n-rk}^{(p,q)}(x,y,r,C)\frac{(q')_k(r^r y)^k}{k!},$

(3.6) $A_n^{(p,q)}(x,y,r,C) = \sum_{k=0}^{n} A_{n-k}^{(p',q')}(x,y,r,C)A_k^{(p-p',q-q')}(x,y,r,C),$

and

(3.7) $A_n^{(p,q)}(x,y,r,C) = \sum_{k=0}^{n} A_{n-k}^{(p-p',q')}(x,y,r,C)A_k^{(p,q-q')}(x,y,r,C)\,.$

4. The Polynomials $\left\{N_n^{(a,b;c,d)}(x,y,r,C)/n = 0,1,2,\ldots\right\}$.

Consider

(4.1) $N_n^{(a,b;c,d)}(x,y,r,C) = \sum_{k=0}^{n} A_k^{(a,b)}(x,y,r,C)F_{n-k}^{(c-k,d)}(2,x,y,r,r,C),$

Therefore

$$\sum_{n=0}^{\infty} N_n^{(a,b;c,d)}(x,y,r,C)\,t^n$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{n} A_k^{(a,b)}(x,y,r,C)\,F_{n-k}^{(c-k,d)}(2,x,y,r,r,C)\,t^n$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A_k^{(a,b)}(x,y,r,C)t^k\,F_n^{(c-k,d)}(2,x,y,r,r,C)\,t^n$$

$$= \sum_{k=0}^{\infty} A_k^{(a,b)}(x,y,r,C)t^k(C-2xt+yt^2)^{C-k}\left[1-\frac{r^r yt^r}{(C-2xt+yt^2)^r}\right]^{-d}$$

$$= \left[1-\frac{r^r yt^r}{(C-2xt+yt^2)^r}\right]^{-d}(C-2xt+yt^2)^c$$

$$\sum_{k=0}^{\infty} A_k^{(a,b)}(x,y,r,C)\left[\frac{t}{(C-2xt+yt^2)^r}\right]^k$$

Hence

(4.2) $\sum_{n=0}^{\infty} N_n^{(a,b;c,d)}(x,y,r,C)t^n$

$$= (C-2xt+yt^2)^{C-a}\left[1-\frac{r^r yt^r}{(C-2xt+yt^2)^r}\right]^{(d-b)}$$

$$= \sum_{n=0}^{\infty} F_n^{(c-a,d-b)}(2,x,y,r,r,C)t^n$$

Thus

(4.3) $N_n^{(a,b;c,d)}(x,y,r,C) = F_n^{(c-a,d-b)}(2,x,y,r,r,C).$

For *a=c,* (4.2) gives

(4.4) $N_n^{(a,b;c,d)}(x,y,r,C)t^n = \left[1-\frac{r^r yt^r}{(C-2xt+yt^2)^r}\right]^{b-d}$

For *b=d,* (4.2) gives

(4.5) $\sum_{n=0}^{\infty} N_n^{(a,b;c,d)}(x,y,r,C)t^n = (C-2xt+yt^2)^{c-a}.$

For *a=c, b=d.*

(4.6) $\sum_{n=0}^{\infty} N_n^{(a,b;c,d)}(x,y,r,C)t^n = 1\,.$

By an appeal to (4.2), we have

$$\sum_{n=0}^{\infty} N_n^{(a+a',b+b';c+c',d+d')}(x,y,r,C)t^n$$

$$= (C-2xt+yt^2)^{c+c'-(a+a')}\left[1-\frac{r^r yt^r}{(C-2xt+yt^2)^r}\right]^{-[(d+d'-(b+b')]}$$

$$= (C-2xt+yt^2)^{c-a}\left[1-\frac{x^r yt^r}{(C-2xt+yt^2)^r}\right]^{-(d-b)}$$

$$(C-2xt+yt^2)\left[1-\frac{r^r yt^r}{(C-2xt+yt^2)^r}\right]^{-(d'-b')}$$

$$= \sum_{n=0}^{\infty} N_k^{(a,b;c,d)}(x,y,r,C)N_{n-k}^{(a',b';c',d')}(x,y,r,C)t^n$$

Hence

(4.7) $N_k^{(a+a',b+b';c+c',d+d')}(x,y,r,C)$

$$= \sum_{k=0}^{\infty} N_k^{(a,b;c,d)}(x,y,r,C) N_{n-k}^{(a',b';c',d')}(x,y,r,C),$$

which can also be generalized in the form

(4.8) $N_k^{(a_1+\cdots+a_n,b_1+\cdots+b_n;c_1+\cdots+c_n,d_1+\cdots+d_n)}(x,y,r,C)$

$$= \sum_{i_1+\cdots+i_m=n} n_{i_1}^{(a_1,b_1;c_1,d_1)}(x,y,r,C) \ldots N_{i_m}^{(a_m,b_m;c_m,d_m)}(x,y,r,C).$$

An appeal to generating relation (4.2) also shows that

(4.9) $N_n^{(a,b;c,d)}(x,y,r,C) = \sum_{k=0}^{\infty} N_{n-k}^{(a,b\prime a',b')}(x,y,r,C) N_k^{(a,b';c,d)}(x,y,r,C).$

5. The Polynomials $\left\{ \boldsymbol{R_n^{(a,b;c,d)}(x,y,r,C)/n = 0,1,2,\ldots} \right\}$.

Consider

(5.1) $R_n^{(a,b;c,d)}(x,y,r,C) = \sum_{k=0}^{n} A_{n-k}^{(a-k,b)}(x,y,r,C) F_k^{(c,d)}(2,x,y,r,r,C).$

Therefore

$$\sum_{n=0}^{\infty} R_n^{(a,b;c,d)}(x,y,r,C)\, w^n$$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{n} A_{n-k}^{(a-k,b)}(x,y,r,C)\, F_k^{(c,d)}(2,x,y,r,r,C)\, w^n$$

$$= \sum_{k=0}^{\infty} F_k^{(c,d)}(2,x,y,r,r,C) w^k \sum_{n=0}^{\infty} A_n^{(a-k,b)}(x,y,r,C)\, w^n$$

$$= \sum_{k=0}^{\infty} F_k^{(c,d)}(2,x,y,r,r,C) w^k \left(\frac{w}{t}\right)^{a-k} (1 - r^r y w^r)^b$$

$$= \left(\frac{w}{t}\right)^a (1 - r^r y w^r)^b \sum_{k=0}^{\infty} F_k^{(c,d)}(2,x,y,r,C) t^k$$

$$= \left(\frac{w}{t}\right)^a (1 - r^r y w^r)^b (C - 2xt + yt^2)^c \left[1 - \frac{r^r y t^r}{(C - 2xt + yt^2)^r}\right]^d$$

$$= \left(\frac{w}{t}\right)^a (1 - r^r y w^r)^b \left(\frac{t}{w}\right)^c [1 - r^r y w^r]^{-d}$$

[Here $\frac{t}{C-2xt+yt^2} = w$]

$$= \left(\frac{w}{t}\right)^{a-c} [1 - r^r y w^r]^{(-d+b)}$$

$$= \sum_{n=0}^{\infty} A_n^{(a-c,b-d)}(x, y, r, C)\, w^n$$

Hence

(5.3) $\sum_{n=0}^{\infty} R_n^{(a,b;c,d)}(x, y, r, C) = A_n^{(a-c,b-d)}(x, y, r, C).$

For $C=a$

(5.4) $\sum_{n=0}^{\infty} R_n^{(a,b;c,d)}(x, y, r, C)\, w^n = [1 - r^r y w^r]^{b-d}$

and for d$=b$

(5.5) $\sum_{n=0}^{\infty} R_n^{(a,b;c,d)}(x, y, r, C)\, w^n = \left(\frac{t}{w}\right)^{c-a}$

$$= \left[\frac{2xw + 1 + \sqrt{(2xw+1)^2 - 4ycw^2}}{2w^2 y}\right]^{c-a}$$

For $c=a$, $d=b$, (5.2) gives

(5.6) $\sum_{n=0}^{\infty} R_n^{(a,b;a,b)}(x, y, r, C)\, w^n = 1.$

Again by an appeal to generating relation (5.2), we derive

(5.7) $R_n^{(a,b;c,d)}(x, y, r, C)$

$$= \sum_{k=0}^{n} R_{n-k}^{(a,b;e,f)}(x, y, r, C) R_k^{(e,f;c,d)}(x, y, r, C),$$

(5.8) $R_n^{(a+a',b+b';c+c',d+d')}(x, y, r, C)$

$$= \sum_{k=0}^{n} R_k^{(a,b;e,f)}(x, y, r, C) R_{n-k}^{(a',b';c',d')}(x, y, r, C),$$

.

An appeal to generating relation (4.2) also shows that

(5.9) $R_n^{(a_1+\cdots+a_m, b_1+\cdots+b_m; c_1+\cdots+c_m, d_1+\cdots+d_m)}(x, y, r, C)$

$$= \sum_{i_1+\cdots+i_m=n} R_{i_1}^{(a_1,b_1;c_1,d_1)}(x, y, r, C) \ldots R_{i_m}^{(a_m,b_m;c_m,d_m)}(x, y, r, C),$$

6. Polynomials $\left\{A_n^{(p,q)}(x, y, r, C)/n = 0, 1, 2, \ldots\right\}$.

Consider

(6.1) $\sum_{k=0}^{n} A_k^{(p,q)}(x, y, r, C)\, M_{n-k}^{(p-k,q)}(2, x, y, z, r, r, C) = 0$

and

(6.2) $A_0^{(p,q)}(x,y,r,C)=1$

Therefore,

$$1=\sum_{n=0}^{\infty}t^n\sum_{k=0}^{n}A_n^{(p,q)}(x,y,r,C)\,M_{n-k}^{(p-k,q)}(2,x,y,z,r,C)$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{n}A_k^{(p,q)}(x,y,r,C)\,M_n^{(p-k,q)}(2,x,y,z,r,C)\,t^{n+k}$$

$$=\sum_{k_i=0}^{\infty}A_k^{(p,q)}(x,y,z,r,C)t^k\sum_{n=0}^{\infty}M_n^{(p-k,q)}(2,x,y,z,r,r,C)t^n$$

$$=\sum_{k=0}^{\infty}A_k^{(p,q)}(x,y,z,r,C)t^k\,(C-mxt+yt^2)^{p-k}\left[1-\frac{r^r yt^r}{(C-2xt+yt^2)^r}\right]$$

Hence

(6.3) $$(C-2xt+yt^2)^{-p}\left[1-\frac{r^r zt^r}{(C-2xt+yt^2)^r}\right]^q$$
$$=\sum_{k=0}^{\infty}A_k^{(p,q)}(x,y,z,r,C)\frac{t^k}{(C-2xt+yt^2)^k}\ ,$$

Therefore, generating relation for $A_n^{(p,q)}$ is

(6.4) $\left(\frac{w}{t}\right)^p[1-zr^r w^r]^q=\sum_{n=0}^{\infty}A_n^{(p,q)}(x,y,z,r,C)\,w^n$

where

(6.5) $\frac{t}{C-2xt+yt^2}=w.$

The generating relation (6.4) can also be written as

(6.6) $\sum_{n=0}^{\infty}A_n^{(p,q)}(x,y,z,r,C)\,w^n$

$$=\left[\frac{2xw+1+\sqrt{(2xw+1)^2-4ycw^2}}{2w^2y}\right]^{-p}[1-r^r zw^r]^q$$

7. Applications of generating relation (6.6). An appeal to generating relation (6.6) shows that

(7.1) $A_n^{(p+p',q+q')}(x,y,z,r,C)=\sum_{k=0}^{n}A_{n-l}^{(p-q)}(x,y,z,r,C)\,A_k^{(p',q')}(x,y,z,r,C),$

(7.2) $A_n^{(p_1+\cdots+p_m,q_1+\cdots+q_m)}(x,y,z,r,C)$

$$=\sum_{k_1+\cdots+k_m=n}A_{k_1}^{(p,q_1)}(x,y,z,r,C)\ldots A_{k_m}^{(p_m,q_m)}(x,y,z,r,C),$$

(7.3) $A_n^{(p,q-q')}(x,y,z,r,C) = \sum_{k=0}^{[n/r]} A_{n-rk}^{(p,q)} \frac{(q')_q (r^r z)^k}{k!}$

(7.4) $A_n^{(p,q+1)}(x,y,z,r,C) = A_n^{(p,q)}(x,y,z,r,C) - r^r z A_{n-r}^{(p,q)}(x,y,z,r,C),$

(7.5) $A_n^{(p,q)}(x,y,z,r,C) = \sum_{k=0}^{n} A_{n-k}^{(p-p',q-q')}(x,y,z,r,C)\, A_k^{(p',q')}(x,y,z,r,C)$

(7.6) $A_n^{(p,q)}(x,y,z,r,C) = \sum_{k=0}^{n} A_{n-k}^{(p-p',q')}(x,y,z,r,C)\, A_k^{(p,q-q')}(x,y,z,r,C)$

and

(7.7) $A_n^{(p,q)}(x,y,z,r,C) = \sum_{k=0}^{n} A_{n-k}^{(q,p)}(x,y,z,r,C)\, A_k^{(p-q,q-p)}(x,y,z,r,C).$

8. Differentiation and Applications.

Differentiating (6.4) partially with respect to z, we have

(8.1) $\frac{\partial}{\partial z} A_n^{(p,q)}(x,y,z,r,C) = -qr^r A_{n-r}^{(p,q-1)}(x,y,z,r,C).$

Hence by iteration, we obtain

(8.2) $\frac{\partial^m}{\partial z^m} A_n^{(p,q)}(x,y,z,r,C)$

$$= (-1)^m q(q-1)\dots(q-m+1) A_{n-rm}^{(p,q-m)}\left(x,y,\frac{1}{z},r,C\right).$$

Replacing z by $1/z$, we have

(8.3) $\left(z^2 \frac{\partial}{\partial z}\right)^m A_n^{(p,q)}(x,y,z,r,C)$

$$= q(q-1)\dots(q-m+1) A_{n-rm}^{(p,q-m)}\left(x,y,\frac{1}{z},r,C\right).$$

Therefore

(8.4) $e^{t\Omega z}\left\{A_n^{(p,q)}(x,y,z,r,C)\right\}$

$$= \sum_{m=0}^{min([n/r],q)} A_{n-rm}^{(p,q-m)}\left(x,y,\frac{1}{z},r,C\right).\binom{q}{m} r^m t^m$$

where $\Omega \equiv z^2 \frac{\partial}{\partial z}$.

Now making an appeal to

(8.5) $e^{t\Omega z}\{f(z)\} = f\big(z/(1-t)\big),$

we derive from (8.4)

(8.6) $A_n^{(p,q)}(x,y,z-t,r,C) = \sum_{m=0}^{min([n/r],q)} t^m \binom{q}{m} r^{r^m} A_{n-rm}^{(p,q-m)}(x,y,z,r,C).$

Now taking z=0 and replacing t by (–z), we derive

(8.7) $A_n^{(p,q)}(x,y,z,r,C) = \sum_{m=0}^{min([n/r],q)} z^m \binom{q}{m} r^{rm} A_{n-rm}^{(p,q-m)}(x,y,0,r,C),$

which can also be obtained by Maclaurine's series expansion.

9. The Polynomials $\left\{N_n^{(a,b,c,d)}(x,y,r,C)/n = 0,1,2,\ldots\right\}$.

Consider

(9.1) $N_k^{(a,b;c,d)}(x,y,z,r,C) = \sum_{k=0}^{n} A_n^{(a,b)}(x,y,z,r,C)\ M_{n-k}^{(c-k,d)}(x,y,z,r,r,C)$

Therefore,

$$\sum_{n=0}^{\infty} A_n^{(a,b;c,d)}(x,y,z,r,C)t^n$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{n} A_n^{(a,b)}(x,y,z,r,C)\, M_{n-k}^{(p-k,d)}(2,x,y,z,r,r,C)\, t^n$$

$$= \sum_{k=0}^{\infty} A_k^{(a,b)}(x,y,z,r,C)\, t^k \sum_{n=0}^{\infty} M_n^{(c-k,q)}(2,x,y,z,r,r,C)\, t^n$$

$$= \sum_{k=0}^{\infty} A_k^{(a,b)}(x,y,z,r,C)t^k\,(C-2xt+yt^2)^{c-k}$$

$$\left[1 - \frac{r^r z t^r}{(C-2xt+yt^2)^r}\right]^{-d}$$

$$= (C-2xt+yt^2)^c \left[1 - \frac{r^r z t^r}{(C-2xt+yt^2)^r}\right]^{-d}$$

$$\sum_{k=0}^{\infty} A_k^{(a,b)}(x,y,z,r,C)\left[\frac{1}{C-2xt+yt^2}\right]^k$$

Now making an appeal to (6.3), we derive

(9.2) $\sum_{k=0}^{\infty} N_n^{(a,b;c,d)}(x,y,z,r,C)$

$$= (C-2xt+yt^2)^{c-a}\left[1 - \frac{r^r z t^r}{(C-2xt+yt^2)^r}\right]^{-(d-b)}$$

$$= \sum_{n=0}^{\infty} M_n^{(c-a,d-b)}(2,x,y,z,r,r,C)t^n\,,$$

Hence

(9.3) $N_n^{(a,b;c,d)}(x,y,z,r,C) = M_n^{(c-a,d-b)}(2,x,y,z,r,r,C)$

For $a=c$ (9.2) gives

(9.4) $\sum_{n=0}^{\infty} N_n^{(a,b;c,d)}(x,y,z,r,C)t^n = \left[1 - \frac{r^r z t^r}{(C-2xt+yt^2)^r}\right]^{b-d}$

for $d=b$, (9.2) gives

(9.5) $\sum_{n=0}^{\infty} N_n^{(a,b;c,d)}(x,y,z,r,C)t^n = (C-2xt+yt^2)^{c-a}$

Again if $a=c$, $b=d$, we derive from (9.2)

(9.6) $\sum_{n=0}^{\infty} N_n^{(a,b;c,d)}(x,y,z,r,C)t^n = 1$

Further, making an appeal to generating relation (9.2), we establish

(9.7) $N_n^{(a+a',b+b';c+c',d+d')}(x,y,z,r,C)$

$$= \sum_{n=0}^{\infty} N_k^{(a,b;c,d)}(x,y,z,r,C)\, N_k^{(a',b';c',d')}(x,y,z,r,C),$$

which can be further generalized in the form

(9.8) $N_n^{(a_1+\cdots+a_n,b_1+\cdots+b_n;c_1+\cdots+c_n,d_1+\cdots+d_n)}(x,y,z,r,C)$

$$= \sum_{i_1+\cdots+i_m=n} N_{i_1}^{(a_1,b_1;c_1,d_1)}(x,y,z,r,C) \ldots R_{i_m}^{(a_m,b_m;c_m,d_m)}(x,y,r,C).$$

Again starting with generating relation (9.2), we can further derive

(9.9) $N_n^{(a,b;c,d)}(x,y,z,r,C)$

$$= \sum_{k=0}^{\infty} N_{n-k}^{(a,b;a',b')}(x,y,z,r,C)\; N_k^{(a',b';a,b)}(x,y,z,r,C)$$

10. The Polynomials $\left\{\frac{R_n^{(a,b,c,d)}(x,y,z,r,C)}{n} = 0,1,2,3,\ldots\right\}$.

Consider

(10.1) $R_k^{(a,b,c,d)}(x,y,z,r,C)$

$$= \sum_{k=0}^{n} A_{n-k}^{(a-k,b)}(x,y,z,r,C)\; M_k^{(c,d)}(2,x,y,z,r,r,C)$$

Therefore,

$$\sum_{n=0}^{\infty} R_n^{(a,b;c,d)}(x,y,z,r,C)w^n$$

$$= \sum_{k=0}^{\infty}\sum_{n=0}^{\infty} A_k^{(a-k,b)}(x,y,z,r,C)w^n\, M_k^{(c,d)}(2,x,y,z,r,r,C)\, w^k$$

$$= \sum_{k=0}^{\infty} M_k^{(c,d)}(2,x,y,z,r,r,C)\, w^k \left(\frac{w}{t}\right)^{a-k} (1-zr^r w^r)^b$$

[by (6.4)]

$$= \left(\frac{w}{t}\right)^{a} (1 - zr^r w^r)^b \sum_{k=0}^{\infty} M_k^{(c,d)}(2, x, y, z, r, r, C)\, t^k$$

$$= \left(\frac{w}{t}\right)^{a} (1 - zr^r w^r)^b (C - 2xt + yt^2)^c \left[1 - \frac{r^r z t^r}{(C - 2xt + yt^2)^r}\right]^{-d}$$

$$= \left(\frac{w}{t}\right)^{a} (1 - zr^r w^r)^b \left(\frac{t}{w}\right)^{c} [1 - zr^r w^r]^{-d}$$

Therefore

(10.2) $\sum_{n=0}^{\infty} R_n^{(a,b;c,d)}(x, y, z, r, C) w^n = \left(\frac{w}{t}\right)^{a-c} (1 - zr^r w^r)^b$

which by an appeal to (9.6) gives

(10.3) $R_n^{(a,b;c,d)}(x, y, z, r, C) = A_n^{(a-c,b-d)}(x, y, z, r, C)$

For a=c, (10.2) gives

(20.4) $\sum_{n=0}^{\infty} R_n^{(a,b;a,d)}(x, y, z, r, C) w^n = [1 - zr^r w^r]^{b-d}$

and for b=d, (10.2) gives

(10.5) $\sum_{n=0}^{\infty} R_n^{(a,b;c,b)}(x, y, z, r, C) w^n = \left(\frac{t}{w}\right)^{c-a}$

$$= \left[\frac{2xw + 1 + \sqrt{(2xw + 1)^2 - 4ycw^2}}{2w^2 y}\right]^{c-a}$$

while for c=a, d=b, (10.2) gives

(10.6) $\sum_{n=0}^{\infty} R_n^{(a,b;c,d)}(x, y, z, r, C) w^n = 1$

Making an appeal to generating relation (10.2), we derive

(10.7) $R_n^{(a,b;c,d)}(x, y, z, r, C)$

$$= \sum_{k=0}^{n} R_{n-k}^{(a,b;e,f)}(x, y, z, r, C)\, R_k^{(e,f;c,d)}(x, y, z, r, C),$$

and

(10.8) $R_n^{(a+a',b+b';c+c',d+d')}(x, y, z, r, C)$

$$= \sum_{k=0}^{n} R_{n-k}^{(a,b;c,d)}(x, y, z, r, C)\, R_k^{(a',b';c',d')}(x, y, z, r, C).$$

Here (10.8) may be further generalized in the form:

(10.9) $R_n^{(a_1+\cdots+a_m,b_1+\cdots+b_m;c_1+\cdots+c_m,d_1+\cdots+d_m)}(x, y, z, r, C)$

$$= \sum_{i_1+\cdots+i_m=n} N_{i_1}^{(a_1,b_1)}(x,y,z,r,C) \dots R_{i_m}^{(a_m,b_m)}(x,y,r,C).$$

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